# ELEC 201 Videos

## Charge/Current/Voltage

### Charge

$q_e = -1.602 \times 10^{-19}$ C

$q_p = 1.602 \times 10^{-19}$ C

q

-measured in Coulombs (C)

$q = \int_{t_1}^{t_2} i \, dt$

### Current

i

-measured in A, C/s

ampere = C/s

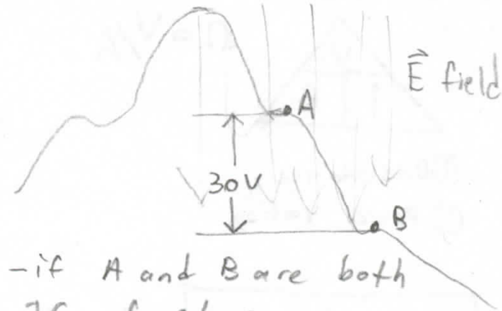
### Voltage

V

-measured in Volts

-Electric height

Volt = J/C




-if A and B are both 1C of charge

-A→B, delivers 30J of energy

- B→A, costs 30J of energy

### Average Current

q(t)

- How many net C of charge have passed the checkpoint to the right at a given time

- $i_{avg} = \frac{\Delta q}{\Delta t}$

### instantaneous Current

$i(t) = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$

$i = \frac{dq}{dt}$

### Energy

W

-measured in Joules

$W = \int_{t_1}^{t_2} Vi \, dt$

### Power

P

-measured in Watts

Watt = J/S

$P = Vi$

$P = \frac{dw}{dt}$ ; $t = \frac{W}{Vi}$

$W = \int_{t_1}^{t_2} p \, dt$

Current flowing:

Uphill → Delivering power

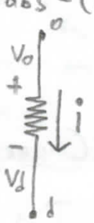
downhill → absorbing power

### Tellegens thereom

$\sum P_{abs} - \sum P_{del} = 0$

- Total absorbed power equals total delivered power in a circuit

$P_{abs} = (V_o - V_d)\, i$




## Circuits

### Branch

-a link between two proper nodes

-may have zero or more binary nodes

### Binary Node

-Does not affect the shape of the circuit

### Atomic Branch

-Branch with no binary nodes

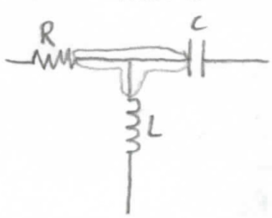
### Branch

-The path that joins two true nodes. Atomic if no binary nodes inside

### Binary Node


### (true) Node

-3 or more




### Series

- Elements in the same branch are in series

### Parallel

① Both elements are in atomic branches

② Both atomic branches share the same true nodes. Must share both

### Series/Parallel

-Two elements in series have the same current.

But two elements that have the same current aren't necessarily in series

-Two elements in parallel have the same voltage but not the other way

### Circuit with no true Nodes

-Promote one B-node to rank of true node (arbitrary)

### Active Element

Battery, generator, etc, sources

Deliver power (usually)

### Passive Element

Resistor, inductor, Capacitor

Absorb power (Sometimes)

### Resistance

- resists Currents

i

### Inductance

-resists changes in Currents $\frac{di}{dt}$

### Capacitance

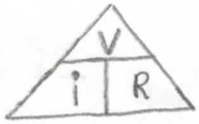
-Resists changes in Voltages $\frac{dV}{dt}$

### Passive element

-Assume water falls from the sky

## Ohm's Law

-R in ohms ($\Omega$)




$\Omega = V/A$

① Resistance
② Power rating

## Inductor

-L in Henrys (H)

$V = L\frac{di}{dt}$

① Inductance
② Current rating

## Capacitor

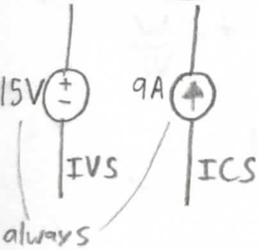
-C in Farads (F)

$i = C\frac{dV}{dt}$ ; $V = \frac{q}{C}$

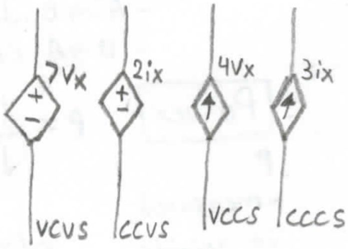
① Capacitance
② Voltage rating

## Active (Sources)

Independent:




Dependent:




## Kirchoff's Current Law

$\sum_{node} i_{in} = \sum_{node} i_{out}$ $\sum_{Gauss} i_{in} = \sum_{Gauss} i_{out}$

$\sum_{Bndry} i_{in} = \sum_{Bndry} i_{out}$

## Super Node

-gaussian surface that may cover many nodes

$\sum_{loop} V_{up} = \sum_{loop} V_{down}$

## Kirchoff's Voltage Law

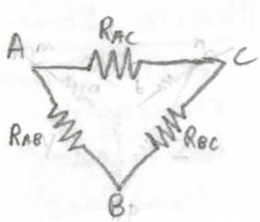
-a loop is a closed path that only crosses a node once, ie: arrive in it and leave out of it

## Window pane (Mesh)

-a loop that does not contain any loops inside

## Neither in Series/Parallel

Triangle/Delta/Pi Config:




## Addition Simplification

$R_{eq} = R_1 + R_2 \ldots + R_n$ SERIES

$L_{eq} = L_1 + L_2 \ldots + L_n$ SERIES

$V_{source} = V_1 + V_2 \ldots + V_n$ SERIES

$C_{eq} = C_1 + C_2 \ldots + C_n$ PARALLEL

$i_{source} = i_1 + i_2 \ldots + i_n$ PARALLEL

## Inverse Simplification

$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} \ldots + \frac{1}{R_n}\right)^{-1}$ PARALLEL

$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} \ldots + \frac{1}{L_n}\right)^{-1}$ PARALLEL

$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} \ldots + \frac{1}{C_n}\right)^{-1}$ SERIES

*midterm demonstrate not on slides

## Inverses

Conductance:

$G = \frac{1}{R}$ -measured in Siemens

Inductance$^{-1}$:

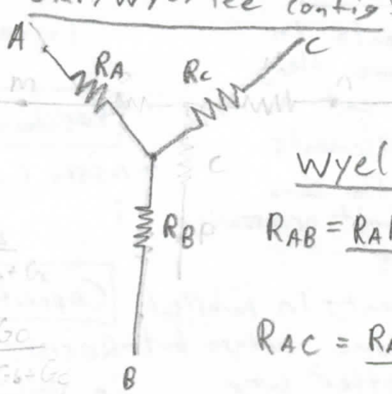
$\Gamma = \frac{1}{L}$ -measured in $H^{-1}$

gamma

Capacitance$^{-1}$:

$T = \frac{1}{C}$ -measured in $F^{-1}$

Star/Wye/tee config:




$\Delta$Delta $\rightarrow$ Wye(Y):

$R_A = \frac{R_{AB}R_{AC}}{R_{AB}+R_{AC}+R_{BC}}$

$R_B = \frac{R_{AB}R_{BC}}{R_{AB}+R_{AC}+R_{BC}}$

$R_C = \frac{R_{AC}R_{BC}}{R_{AB}+R_{AC}+R_{BC}}$

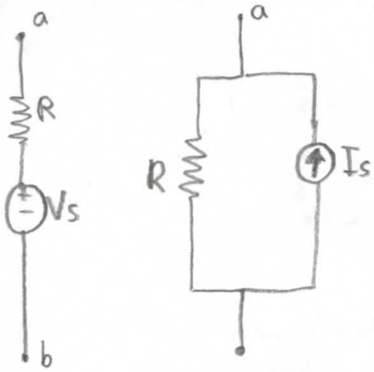
Wye(Y) $\rightarrow$ $\Delta$Delta:

$R_{AB} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_C}$

$R_{AC} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_B}$

$R_{BC} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_A}$

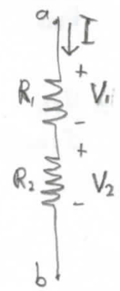
## Source Transformations




$V_S = R I_S$

$I_S = \frac{V_S}{R}$

## Voltage Divider




$V_1 = R_1 I$

$V_2 = R_2 I$

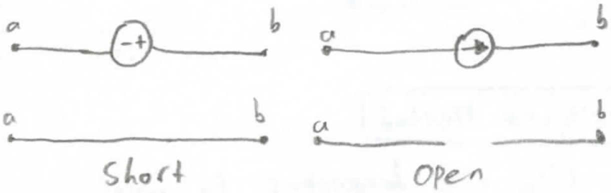
$V_{ab} = V_2 + V_1$

$V_{ab} = I(R_2 + R_1)$

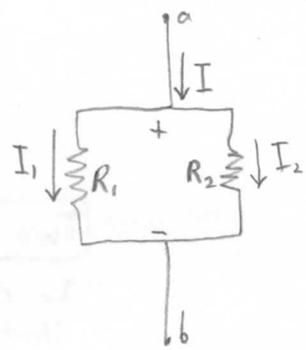
$\frac{V_1}{V_{ab}} = \frac{R_1}{R_{eq}}$ "Same %"

## Superposition

killing a Vsource: killing a Isorce:




## Current Divider




$I_1 = \frac{V_{ab}}{R_1}$

$I_2 = \frac{V_{ab}}{R_2}$

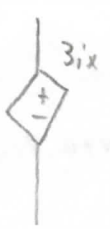
$\frac{I_1}{I} = \frac{R_2}{R_1 + R_2}$ "Same %"

$\frac{I_1}{I} = \frac{G_1}{G_1 + G_2}$

$I = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_{ab}$

## Modified Nodal Analysis

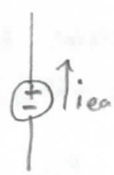
Controlling Variable:




- 3ix is a controlling variable

Evil currents:

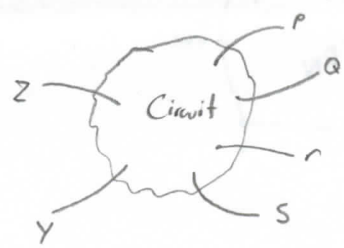
- currents in evil branches




## Solution Method

1. Choose REF node
2. Choose R/RV current directions
3. Label every true node (KCL)
4. Label every evil current (EVL)
5. Label every controlling variable (CTL)

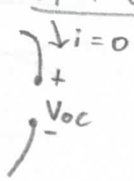
## Thenevin and Norton Equivilants



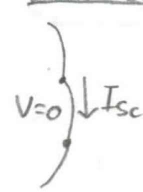

Two nodes = one port (Could be binary or true)

## (4) New ideal Circuit Elements

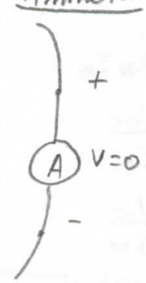
Open circuit:



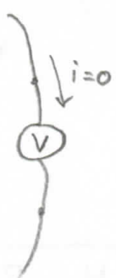

Short circuit:



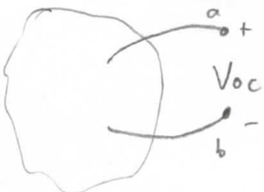

Ammeter:



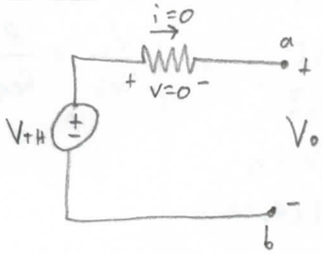

Voltmeter:



## Classic Method




Open Circuit test:

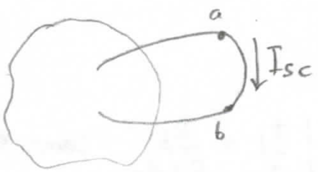


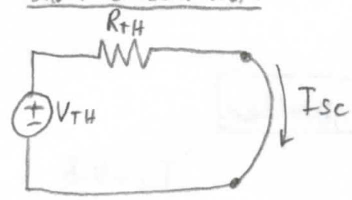

$I_{sc} = \frac{V_{TH}}{R_{TH}}$

$R_{TH} = \frac{V_{TH}}{I_{sc}}$

$V_{TH} = V_{oc}$

Short circuit test:



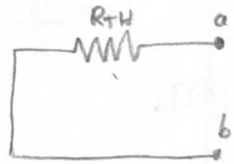




## Kill Sources (All independent)

- Find $R_{eq} = R_{TH}$

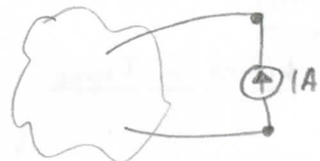
## Only controlled Sources

$V_{oc} = 0$



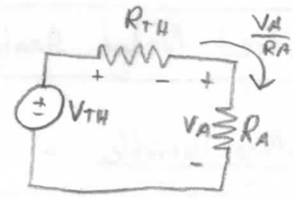

How.... :

apply a 1Amp current between the nodes, solve for RTH




## Two Resistors Method

"In real life its dangerous to apply Short circuit test"


"Do this again with a second Resistor $R_B$"

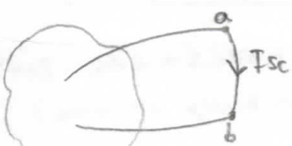
SOLVE for $V_{TH}, R_{TH}$

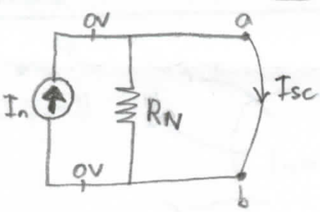
$V_{TH} - R_{TH}\frac{V_A}{R_A} = V_A$

$V_{TH} - R_{TH}\frac{V_B}{R_B} = V_B$

## Norton Mayer Equivilant

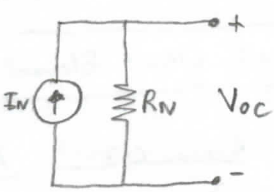
Short circuit test:




$I_N = I_{sc}$

Open circuit test:

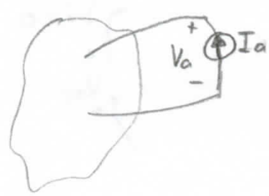



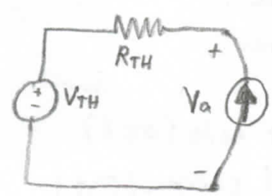

$V_{oc} = R_N I_N$

$R_N = \frac{V_{oc}}{I_n}$

$R_n = \frac{V_{oc}}{I_{sc}}$

## 1A/2A (UBC) Method




"Repeat with new Isource

$V_{th} + R_{th} I_a - V_a = 0$

$V_{th} + R_{th} I_b - V_b = 0$

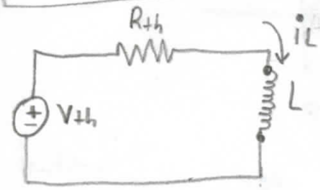
## Max Power Criterion

$P_{MAX} = \frac{V_{th}^2}{4R_{th}}$

$P_{MAX} = \frac{I_{sc}^2 R_{th}}{4}$

# First order Circuits

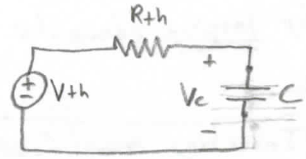
## RL Circuit




DE: $L\frac{di_L}{dt} + Ri_L = V_s = 0$

sol'n: $i_L(t) = \left(I_0 - \frac{V_s}{R}\right)e^{-t/(\frac{L}{R})} + \frac{V_s}{R}$

## RC Circuit




DE: $RC\frac{dV_c}{dt} + V_c = V_{th}$

Sol'n: $V_c(t) = (V_{c_0} - V_{th})e^{-t/RC} + V_{th}$

## Impedance

$Z_L = Lp$ $\quad * \; p = \frac{d}{dt} \; (s^{-1})$

$Z_C = \frac{1}{Cp}$

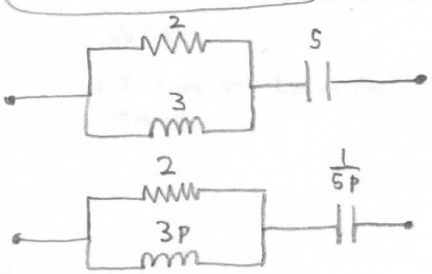
$Z_r = R$

General Ohm's Law:

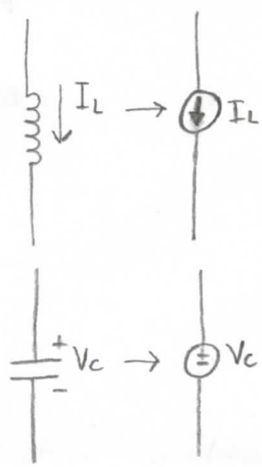
$V = Z \cdot i$

volts, impedance, current

## R/L/C Impedance




Right after we move a switch $t = 0^+$




### Capacitor

where $V_0$ is initial voltage

$V_C = V_0 e^{-t/RC}$

$Q = CV_0 e^{-t/RC}$

$I = \frac{V_0}{R_{th}} e^{-t/RC}$

### Inductor

where $V_b$ is final voltage

$V_L = V_b e^{-tR/L}$

$I = \frac{V_b}{R}(1 - e^{-tR/L})$

$V_c(t) = (V_{co} - V_{cf})e^{-t/RC} + V_{cf}$

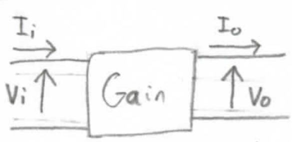
$I_L(t) = (I_{Lo} - I_{Lf})e^{-tR/L} + I_{Lf}$

where $V_{co}$ and $I_{Lo}$ are DCSS of both

$V_{cf} = V_{TH}$ seen by capacitor

$I_{Lf} = I_N$ seen by inductor

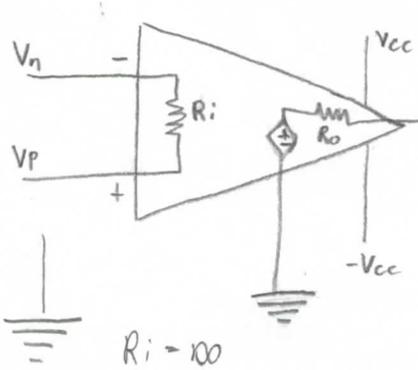
## Operational Amplifiers

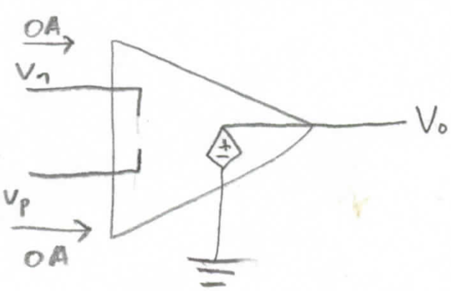



gains

$A_V = \frac{V_o}{V_i}$ (V/V) $\quad A_I = \frac{I_o}{I_i}$ (A/A) $\quad A_P = \frac{P_o}{P_i}$ (W/W)

$A_V^{DB} = 20\log_{10}\left|\frac{V_o}{V_i}\right| \quad A_I^{DB} = 20\log_{10}\left|\frac{I_o}{I_i}\right| \quad A_P^{DB} = 10\log_{10}\left|\frac{P_o}{P_i}\right|$




$R_i = 100$

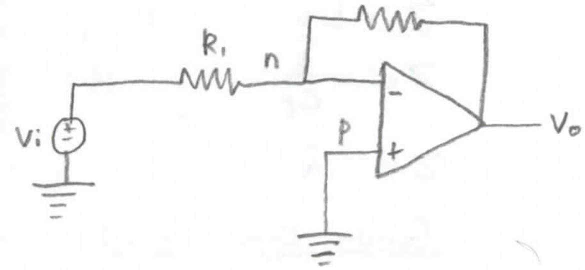
$R_o = 0$

$A = 100$ (voltage gain)

$V_o = A(V_p - V_n)$

## Negative Feedback

-connecting the output to the inverting input by passive elements




## Inverting amplifier

$V_p = V_n$

$V_p = 0$

- Don't write a kcl for $V_o$**

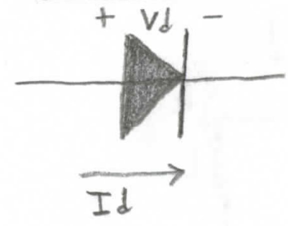
$V_p = V_n$

-as long as no saturation

$$n: \frac{V_i - V_n}{R1} = 0 + \frac{V_n - V_o}{R_2}$$

$$\therefore V_o = -\frac{R_2}{R_1} V_i$$

## Ideal Diode




$I_d = I_o e^{V_D/V_T}$ constant

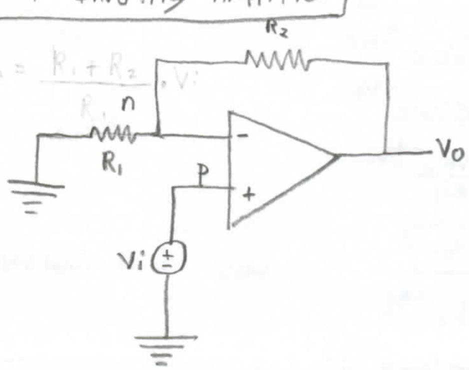
## Saturation

$-V_{cc} \leq V_o \leq +V_{cc}$

Positive Saturated $\rightarrow V_o = +V_{cc}$, $V_p > V_n$

Negative Saturated $\rightarrow V_o = -V_{cc}$, $V_p < V_n$

## Non-Inverting Amplifier




$V_p = V_n$

$V_p = V_i$

$$\therefore V_o = \frac{R_1 + R_2}{R_1} \cdot V_i$$

$$n: \frac{V_o - V_n}{R_2} = 0 + \frac{V_n}{R_1}$$

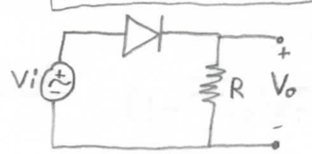
# Rectifier Circuits

Transformer — Rectifier — Filter — Voltage Rectifier — Load

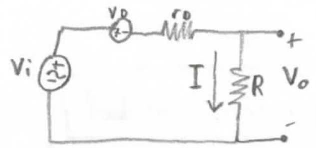
## Types of Rectifiers

1. Half-Wave
2. Full-Wave:
   - Center tap
   - Bridge type

## Half-Wave Rectifier



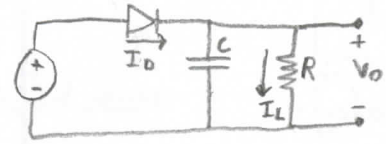

↓ Equivilant




$$I = \frac{V_i - V_D}{R + r_D}$$

$$V_o = R\left(\frac{V_i - V_D}{R + r_D}\right)$$

## Rectifier Circuit with resistive load




For Half Wave:

$$V_r = \frac{V_p T}{RC} = \frac{V_p}{fRC} = \frac{I_L}{fC}$$

For Full wave:

$$V_r = \frac{V_p T}{2RC} = \frac{V_p}{2fRC} = \frac{I_L}{2fC}$$

Load Voltage:

$$V_L = V_p - \frac{1}{2}V_r$$

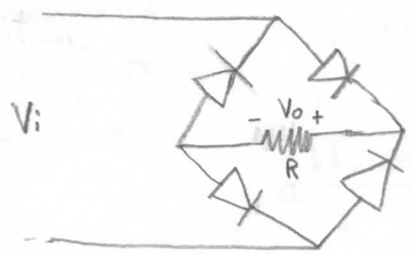
When ripple is small

$$I_L = \frac{V_p}{R_L}$$

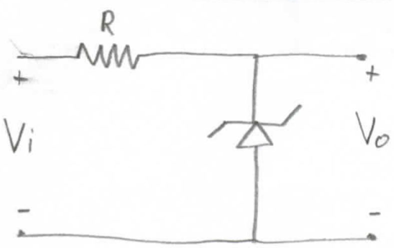
Load Current:

$$I_L = \frac{V_L}{R_L}$$

## Full Wave Bridge Rectifier




## Zener diodes as Shunt Voltage regulators



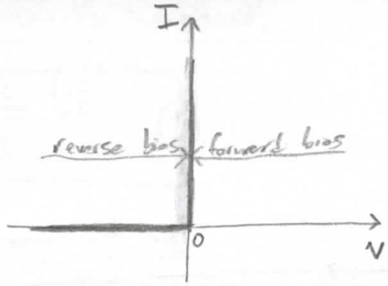

$$\text{Line regulation} = \frac{\Delta V_o}{\Delta V_{in}}$$

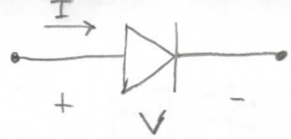
$$\text{Load regulation} = \left|\frac{\Delta V_o}{\Delta I_L}\right|$$

# Diodes

## Ideal diode


- Allows current to flow in one direction
- No voltage drop across terminals when allowing current to flow




(2) modes of operation:

1. Reverse bias, $V<0$, $I=0$, behaves like open circuit
2. Forward bias: $V=0$, $I>0$, behaves like short circuit

## Real diode

- Allows a small amount of reverse current
- Has a small voltage drop when current flowing

$$i = I_S\left(e^{\frac{V}{nV_T}}-1\right)$$

where: $\boxed{i}$ = current through the diode

$\boxed{V}$ = Voltage across the diode

$\boxed{I_S}$ = reverse saturation current, very small usually $10^{-12}A$ or $10^{-15}A$

$\boxed{n}$ = empirical constant

$\boxed{V_T}$ = Thermal voltage

$$V_T = \frac{kT}{q}$$

where:

$\boxed{k}$ = Boltzmann's constant

$\boxed{T}$ = absolute temperature

$\boxed{q}$ = base charge

Simplified eqn:

- Because usually $i \gg I_S$, $v \gg 10nV_T$

$$i \approx I_S e^{\frac{V}{nV_T}}$$

## Models

1. Ideal Model
2. Exponential Model
3. Constant Voltage Drop
4. Piecewise linear
5. Small signal Model

## Ideal Model

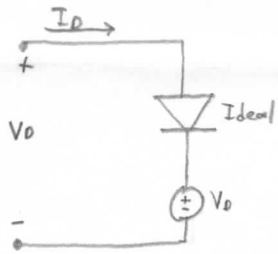
- Try all possibilities $2^n$ possibilities where $n$ is the number of diodes

## Equation Model

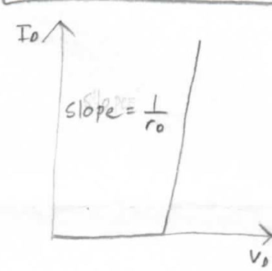
- Make a system of equations
- Most accurate
- Hard without a calculator
- Also called the exponential model

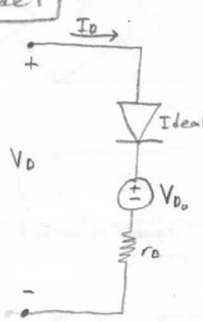
## Constant Voltage Drop

- The diode can be replaced with an ideal diode and voltage source in series




## Piecewise Linear Model




$$\frac{1}{r_D} = \frac{1mA - 0mA}{0.7V - V_{D0}}$$

- To find $V_{D0}$

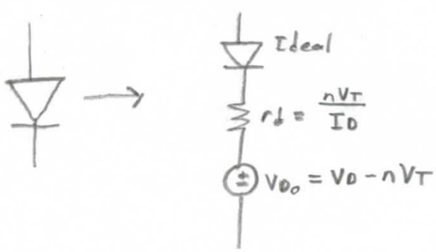
## Small Signal Model

1. Solve the operating point of the diode in the DC circuit.

To get DC:

- Short all AC voltage sources
- Open all AC current sources
- Short all inductors
- Open all capacitors

2. Replace diode with its small signal equivilant in AC circuit

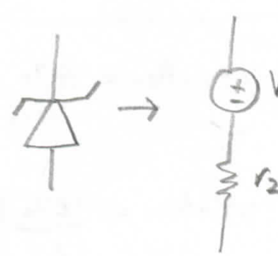



To get AC:

- Short all DC voltage sources
- open all DC current sources
- open all inductors
- Short all capacitors

## Zener diode

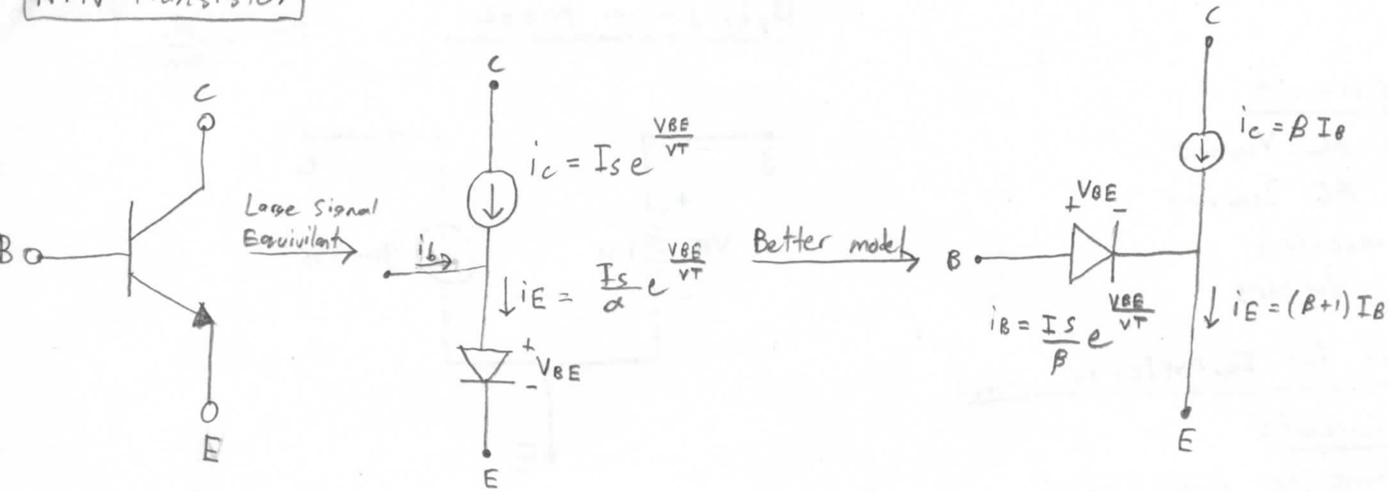
- allows current to flow in reverse at certain voltage



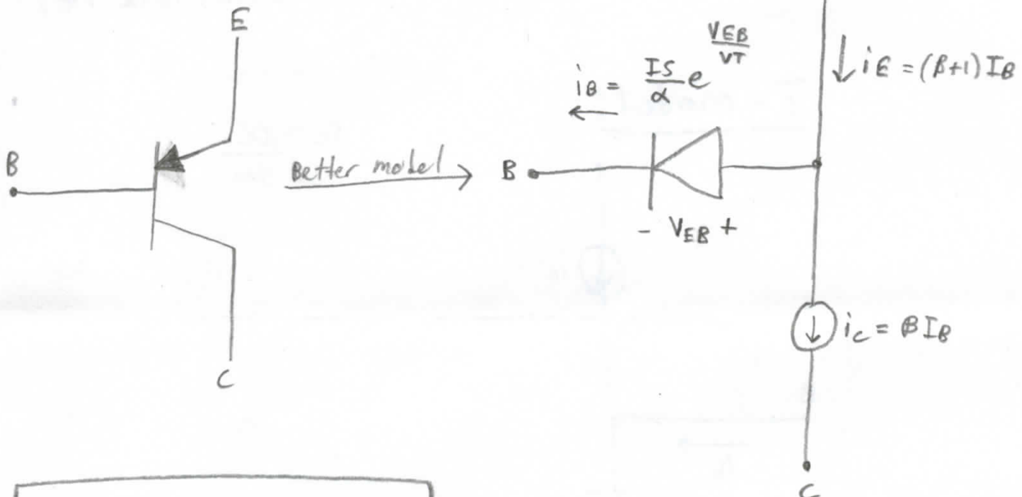

$$V_{Z0} = V_Z - r_Z I_Z$$

# BJT's

## NPN transistor



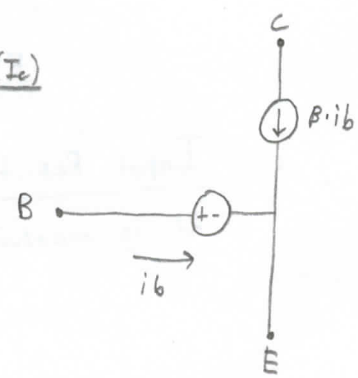

## PNP Transistor



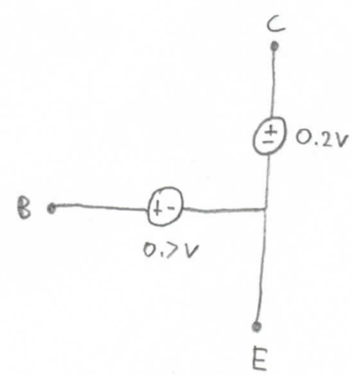

* only work in <u>active</u> region

## Modes of Operation

| BJT Mode | | Collector Current ($I_c$) |
|---|---|---|
| Cutoff | $\longrightarrow$ | $i_c = 0$ |
| Active | $\longrightarrow$ | $i_c = \beta \cdot I_B$ |
| Saturation | $\longrightarrow$ | $i_c < \beta \cdot I_B$ |




ACTIVE




SATURATION

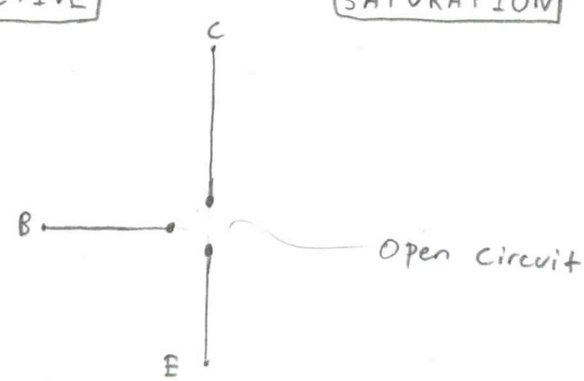
### Relationships

$I_E = I_B + I_c$

$I_c = \beta \cdot I_B$

$I_c = \alpha \cdot I_E$

$\alpha = \frac{\beta}{\beta+1}$ $\quad \beta = \frac{\alpha}{1-\alpha}$



# BJT Amplifiers

## Steps:

① **Get DC circuit:**
- Short all AC Vsources
- Open all AC Isources
- open all capacitors
- short all inductors

② **Solve circuit for $I_c$, $r_\pi$ (or $r_e$), $g_m$:**

③ **Get AC Circuit:**
- replace transistor with hydrid-π circuit
- Short all DC Vsources
- Open all DC Isources
- Short all capacitors
- Open all inductors

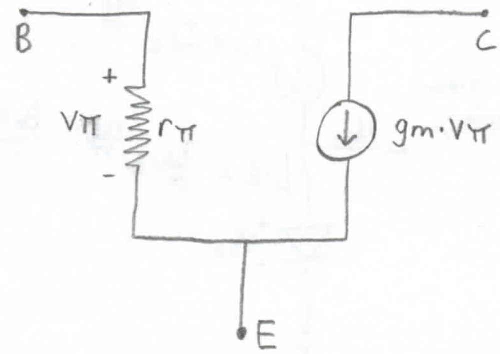
④ **Solve AC circuit for (gain)**

# Models

$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \frac{\beta}{g_m}$$

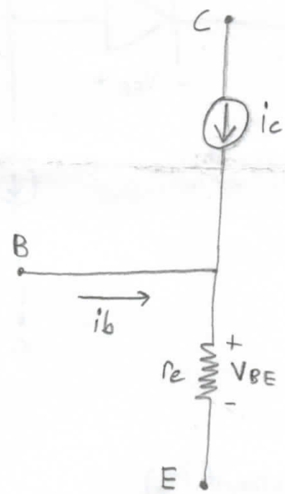
## Hybrid-π model:




Input Resistance ($R_i$) → use hydrid-π model if $R_i$ is measured from the base. (use $r_\pi$)

## T-model:

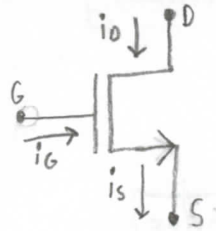
$$r_e = \frac{\alpha}{g_m}$$




Input Resistance ($R_i$) ⟶ use T-model if $R_i$ is measured from the base. (use $r_e$)

# MOSFET

## Enhancement N-MOSFET




**Triode Region** $V_{DS} < V_{GS} - V_t$

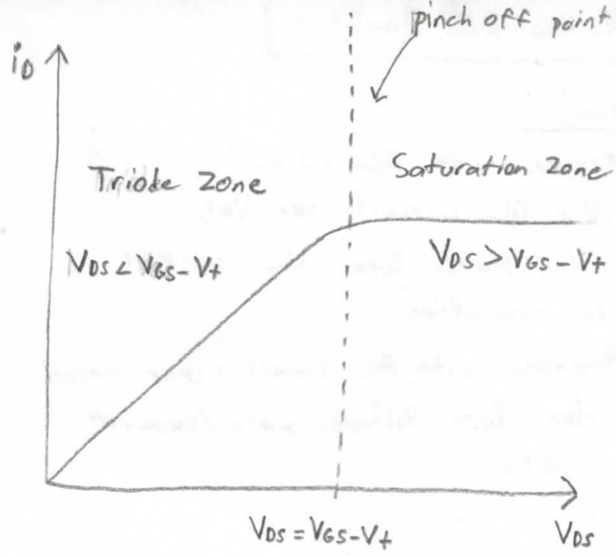
$$i_D = i_S = k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

**Saturation Region** $V_{DS} > V_{GS} - V_t$

$$i_D = i_S = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2$$

**Cutoff Region** $V_{GS} < V_t$

$$i_D = 0$$




**Small $V_{DS}$ Assumption** $V_{DS} < 0.2V$

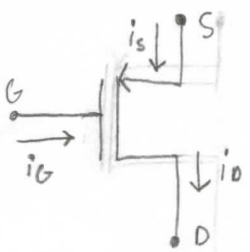
* in Triode Region

$$i_D \approx k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS}\right]$$

Where: $i_D \approx \frac{1}{r_{DS}} \cdot V_{DS}$

$$r_{ds} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_t)}$$

## Enhancement P-MOSFET




**Triode Region** $|V_{DS}| < |V_{GS} - V_t|$

$$i_D = i_S = k_p' \frac{W}{L}\left[(|V_{GS} - V_t|)|V_{DS}| - \frac{1}{2}V_{DS}^2\right]$$

**Saturation Region** $|V_{DS}| > |V_{GS} - V_t|$

$$i_D = i_S = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2$$

• With Early Voltage:

$\lambda = \frac{1}{V_A}$ where $V_A \neq \infty$

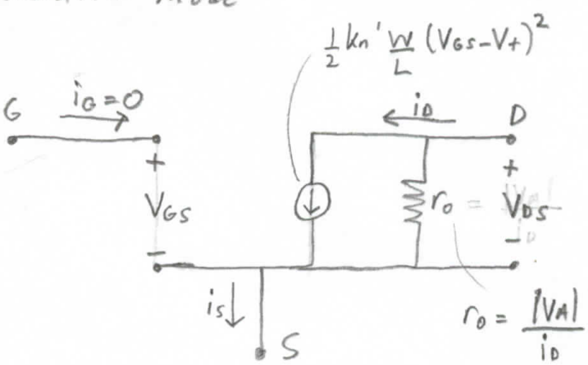
$$i_D = i_S = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2(1 + |\lambda||V_{DS}|)$$

**Cutoff Region** $|V_{GS}| < |V_t|$

$$i_D = i_G = i_S = 0$$

## Large Signal Model

* Enhancement N-MOSFET in Saturation mode

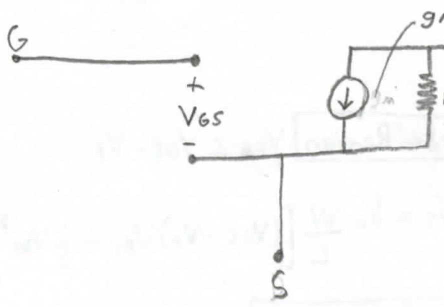


# AC and DC Analysis

Steps:

1. Replace with DC circuit
2. Solve DC circuit for $V_{GS}$, $i_D$, $V_{DS}$. Make sure the MOSFET is in saturation
3. Replace with AC small signal model
4. Solve for voltage gain/current gain, etc

# AC small signal model




$$r_o = \left[\frac{\lambda}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2\right]^{-1} \approx \frac{|V_A|}{i_D}$$

## Formulas

$$\lambda = \frac{1}{V_A}$$

$$g_m = k_n' \frac{W}{L} (V_{GS} - V_t)$$